Section 001 CDS 101 *Introduction to Computational and Data Sciences*

Class 22: Inference and simulations I

April 12, 2018

General

Annoucements

- Questions for Reading 12 due on Friday, April 13th by 5:00pm
  - Introductory Statistics with Randomization and Simulation: from chapter 1, read sections 1.3 (skip 1.3.4), 1.4.1, and 1.5
  - Writeup: An advanced example of a PMF visualization
  - Writeup: Class-size paradox
- Reading for next Tuesday's class: Introductory Statistics with Randomization and Simulation
  - From chapter 2: section 2.3 through to the end of section 2.5
  - From chapter 4: section 4.5 (skip 4.5.3)
- Homework 3 due on Monday, April 16th by 11:59pm.

Case study: Gender discrimination

Study description and data

- In 1972, as a part of a study on gender discrimination, 48 male bank supervisors were each given the same personnel file and asked to judge whether the person should be promoted to a branch manager job that was described as "routine".

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This is an example of an experiment

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Data

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	Promoted	Not Promoted	Total
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% of males promoted: 21 / 24 = 0.875

% of females promoted: 14 / 24 = 0.583

Practice

We saw a difference of almost 30% (29.2% to be exact) between the proportion of male and female files that are promoted. Based on this information, which of the below is true?

1. If we were to repeat the experiment we will definitely see that more female files get promoted. This was a fluke.
2. Promotion is dependent on gender, males are more likely to be promoted, and hence there is gender discrimination against women in promotion decisions.
3. The difference in the proportions of promoted male and female files is due to chance, this is not evidence of gender discrimination against women in promotion decisions.
4. Women are less qualified than men, and this is why fewer females get promoted.

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   Promotion and gender are **dependent**, there is gender discrimination, observed difference in proportions is not due to chance. → **Alternative hypothesis**

A trial as a hypothesis test

- As a process, hypothesis testing is analogous to a court trial
- H_0: Defendant is innocent

  H_A: Defendant is guilty
- We then present the evidence – collect data.

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- Ultimately we must make a decision. How unlikely is unlikely?

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- In a hypothesis test, the burden of proof is on the unusual claim.
- The null hypothesis is the ordinary state of affairs, so it's the alternative hypothesis that we consider unusual and for which we must gather evidence.

Recap: hypothesis testing framework

- We start with a null hypothesis (H_0) that represents the status quo
- We also have an alternative hypothesis (H_A) that represents our research question, i.e. what we're testing for
- We conduct a hypothesis test under the assumption that the null hypothesis is true, either via simulation or theoretical methods
    - If the test results suggest that the data do not provide convincing evidence for the alternative hypothesis, we stick with the null hypothesis
    - If they do, then we reject the null hypothesis in favor of the alternative

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- If the results from the simulations based on the chance model do not look like the data, then we can determine that the difference between the proportions of promoted files between males and females was not due to chance, but **due to an actual effect of gender** (promotion and gender are dependent).

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    - Take out 3 aces $\rightarrow$ there are exactly 13 face cards left in the deck (face cards: A, K, Q, J)
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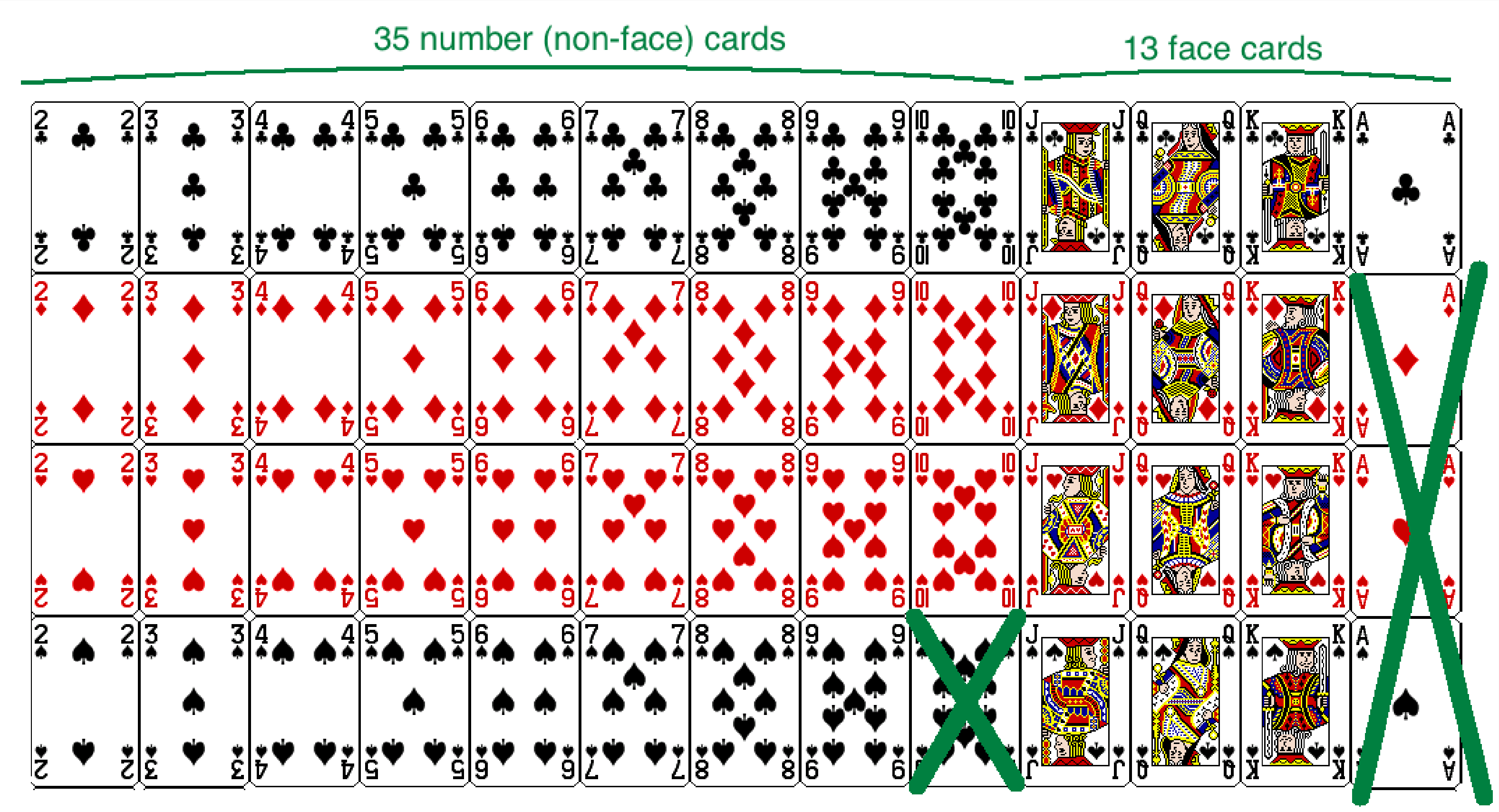
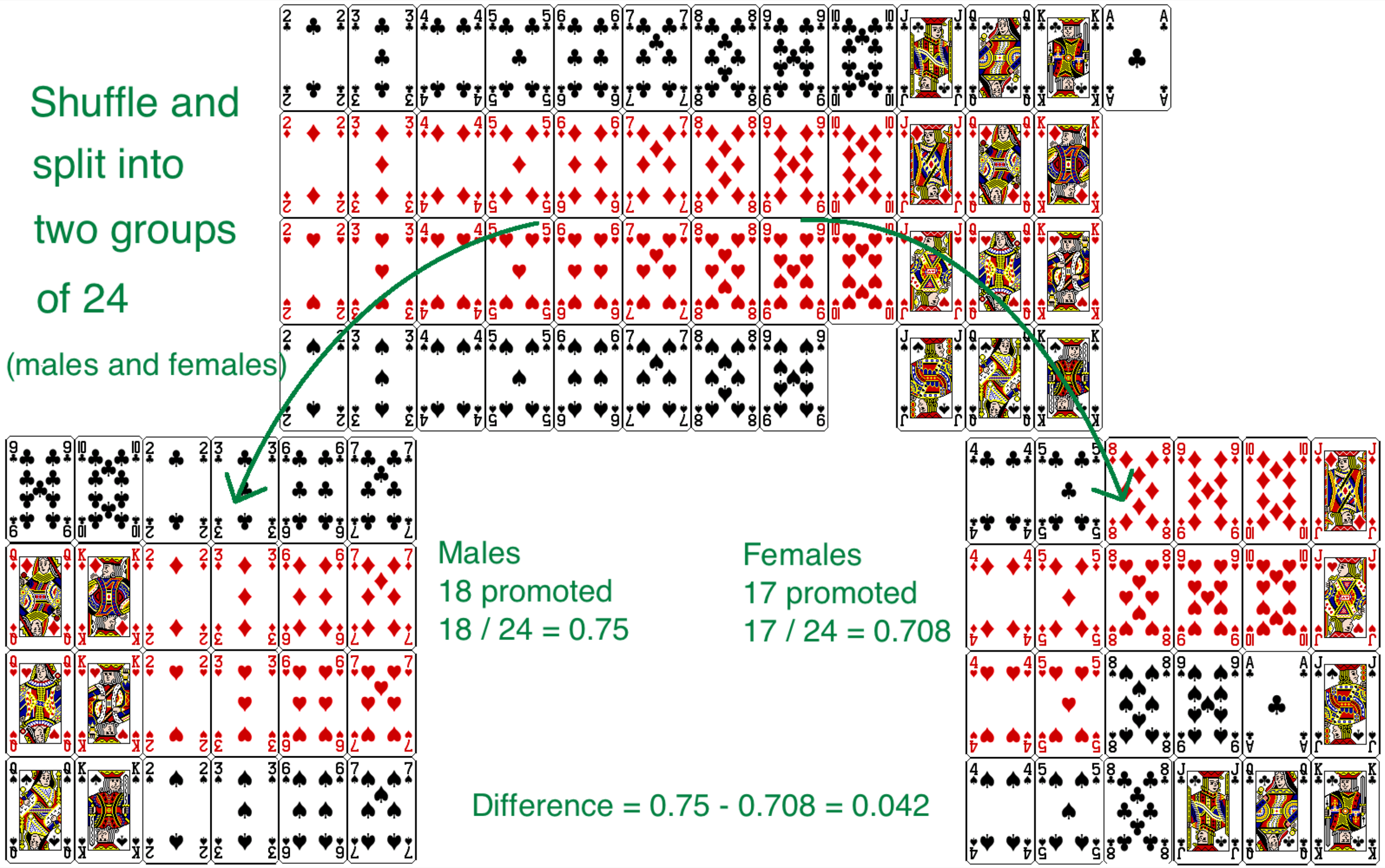
5. Repeat steps 2 – 4 many times

Step 1



Step 2


Shuffle and split into two groups of 24
(males and females)
Males
18 promoted
18 / 24 = 0.75
Females
17 promoted
17 / 24 = 0.708
Difference = 0.75 - 0.708 = 0.042

Simulations in R

Introduce a new package: `infer`

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Download by running:

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Use gender discrimation dataset:

```
applicants <- data_frame(
  sex = c(
    rep("Male", 24),
    rep("Female", 24)),
  outcome = c(
    rep("Promoted", 21),
    rep("Not Promoted", 3),
    rep("Promoted", 14),
    rep("Not Promoted", 10)))
```

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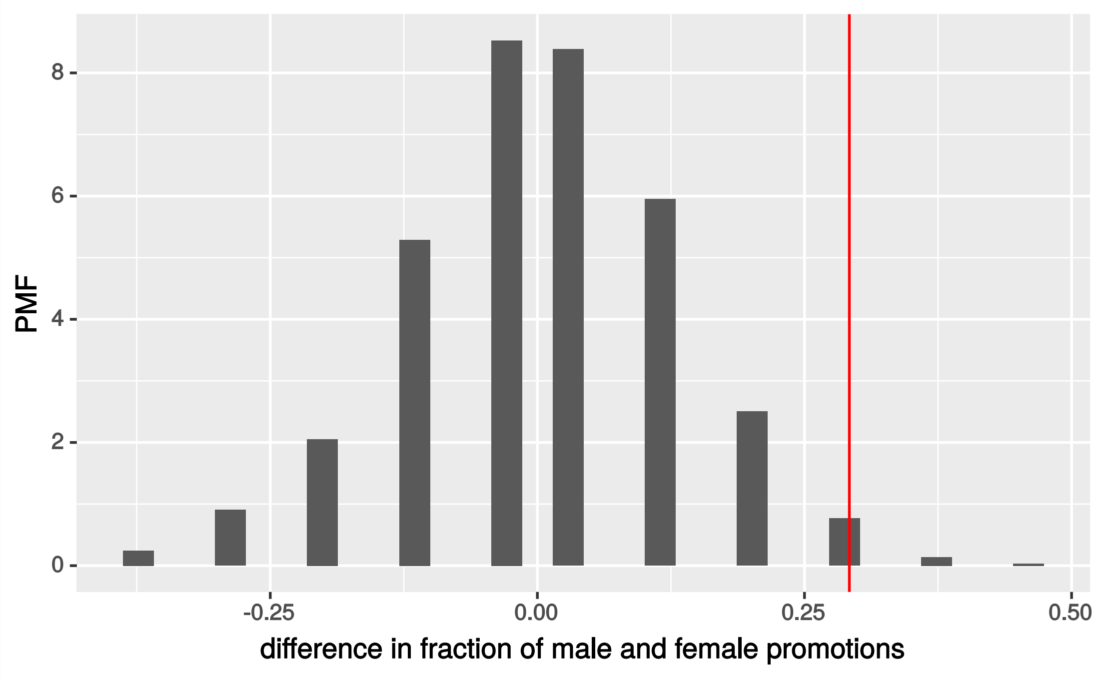
```
simulation_results <- applicants %>%
  specify(outcome ~ sex, success = "Promoted") %>%
  hypothesize(null = "independence") %>%
  generate(reps = 1000, type = "permute") %>%
  calculate(stat = "diff in props", order = c("Male", "Female"))
```

Simulations in R

```
simulation_results %>%
  ggplot() +
  geom_histogram(
    mapping = aes(x = stat, y = ..density..), center = 0) +
  geom_vline(xintercept = 0.875 - 0.583, color = "red") +
  labs(x = "difference in fraction of male and female promotions",
       y = "PMF")
```



Probability of randomly getting result

```
gender_percentiles <- simulation_results %>%
  pull(stat) %>%
  ecdf()

print(1 - gender_percentiles(0.875 - 0.583))
```

```
## [1] 0.005
```

Conclusions from our simulation

Do the results of the simulation provide convincing evidence of gender discrimination against women, i.e. dependence between gender and promotion decisions?

1. No, the data do not provide convincing evidence for the alternative hypothesis, therefore we can't reject the null hypothesis of independence between gender and promotion decisions. The observed difference between the two proportions was due to chance.

2. Yes, the data provide convincing evidence for us to reject the null hypothesis in favor of the alternative hypothesis of gender discrimination against women in promotion decisions. The observed difference between the two proportions was due to a real effect of gender.

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Constructing hypothesis tests

Number of college applications

A survey asked how many colleges students applied to, and 206 students responded to this question. This sample yielded an average of 9.7 college applications with a standard deviation of 7. College Board website states that counselors recommend students apply to roughly 8 colleges. Do these data provide convincing evidence that the average number of colleges all GMU students apply to is *higher* than recommended?

http://www.collegeboard.com/student/apply/the-application/151680.html

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- We start with the assumption the average number of colleges GMU students apply to is 8 (as recommended)

$$H_0 : \mu = 8$$

- We test the claim that the average number of colleges GMU students apply to is greater than 8

$$H_A : \mu > 8$$

Formal testing using p-values

Statistical significance

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- Choose a value for the significance level α (a common choice is 5%)

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In order to evaluate if the observed sample mean is unusual for the hypothesized sampling distribution, we do the following:

- Choose a value for the significance level α (a common choice is 5%)
- Determine the percentile rank of the observed sample mean relative to the null distribution

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- We then use the percentile to calculate the **p-value**, the probability of observing data at least as favorable to the alternative hypothesis as our current data set, if the null hypothesis were true.

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- If the p-value is **higher** than α, we say that it is likely to observe the data even if the null hypothesis were true, and hence **do not reject H_0**.

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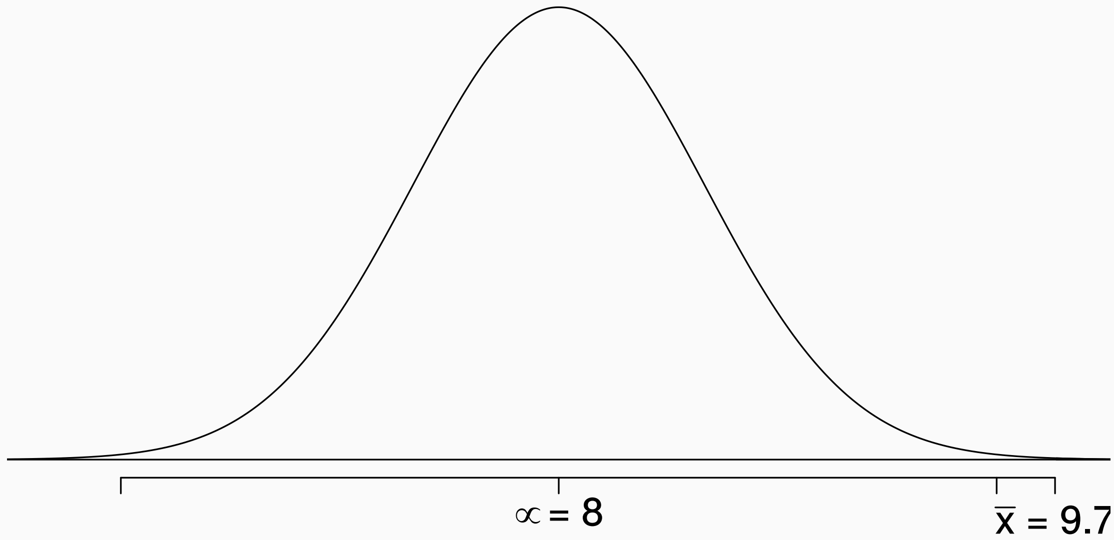

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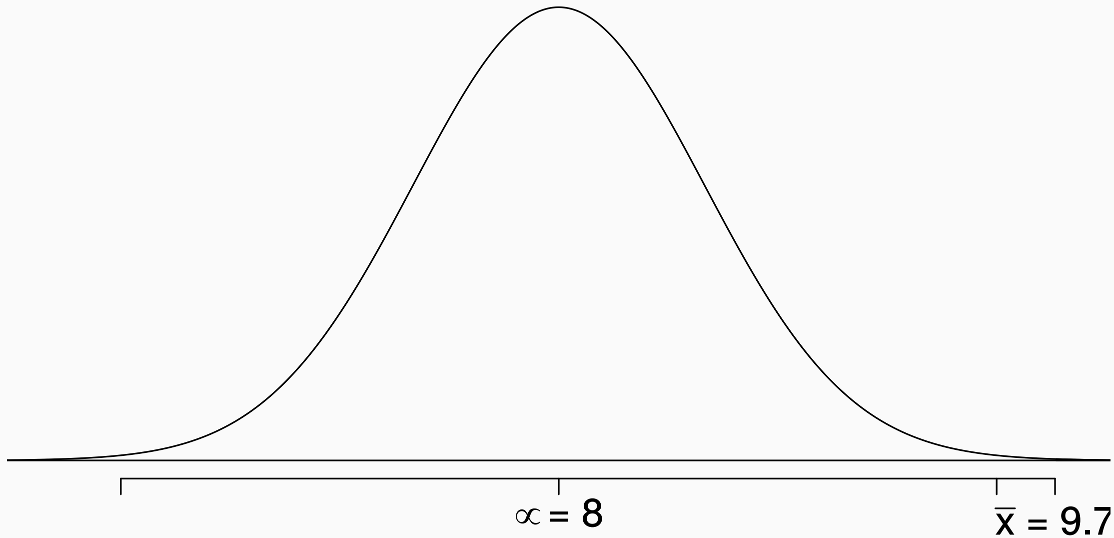


```
1 - pnorm(9.7, mean = 8, sd = 7 / sqrt(206))
```

```
## 0.0002
```

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- The data provide convincing evidence that GMU students apply to more than 8 schools on average.
- The difference between the null value of 8 schools and observed sample mean of 9.7 schools is **not due to chance** or sampling variability.

Example: National Sleep Foundation poll

A poll by the National Sleep Foundation found that college students average about 7 hours of sleep per night. A sample of 169 college students taking an introductory statistics class yielded an average of 6.88 hours, with a standard deviation of 0.94 hours. Assuming that this is a random sample representative of all college students *(probably a bit of stretch)*, a hypothesis test was conducted to evaluate if college students on average sleep *less than* 7 hours per night. The p-value for this hypothesis test is 0.0485. Which of the following is correct?

1. Fail to reject H_0, the data provide convincing evidence that college students sleep less than 7 hours on average.
2. Reject H_0, the data provide convincing evidence that college students sleep less than 7 hours on average.
3. Reject H_0, the data prove that college students sleep more than 7 hours on average.
4. Fail to reject H_0, the data do not provide convincing evidence that college students sleep less than 7 hours on average.
5. Reject H_0, the data provide convincing evidence that college students in this sample sleep less than 7 hours on average.

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Two-sided hypothesis testing with p-values

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$$H_0 : \mu = 7$$
$$H_A : \mu \neq 7$$

Two-sided hypothesis testing with p-values

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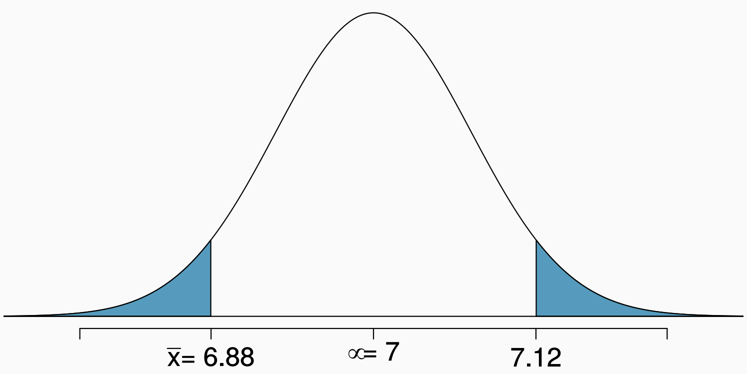
- Hence the p-value would change as well:



$$\begin{aligned} \text{p-value} &= 0.0485 \times 2 \\ &= 0.097 \end{aligned}$$

Credits