

Class 26: Modeling II

April 26, 2018



General

Announcements

- **Homework 4** due on Friday, April 27th by 11:59pm
- Homework 5 (to be posted) will be due on Friday, May 4th by 11:59pm
- Start thinking about your Final Portfolios, due Friday, May 11th by 11:59pm

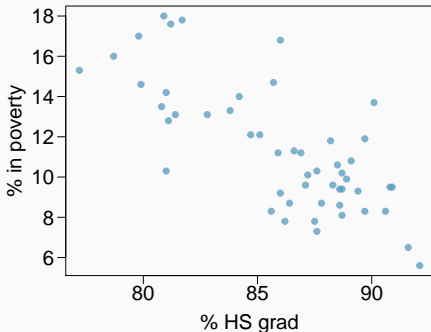
Line fitting, residuals, and correlation

Modeling numerical variables

In this unit we will learn to quantify the relationship between two numerical variables, as well as modeling numerical response variables using a numerical or categorical explanatory variable.

Poverty vs. HS graduate rate

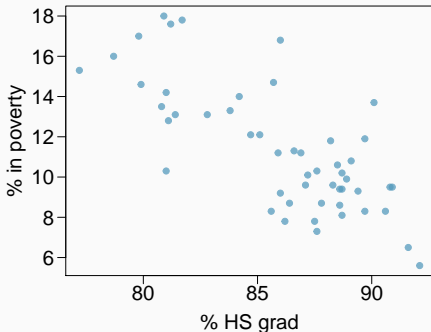
The *scatterplot* below shows the relationship between HS graduate rate in all 50 US states and DC and the % of residents who live below the poverty line (income below \$23,050 for a family of 4 in 2012).



Response variable?

Poverty vs. HS graduate rate

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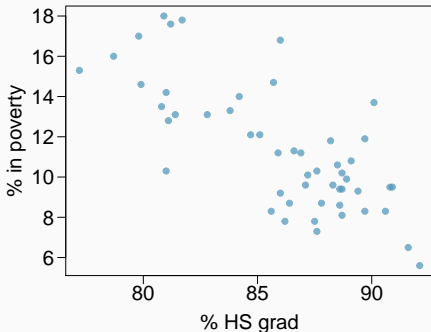


Response variable?

% in poverty

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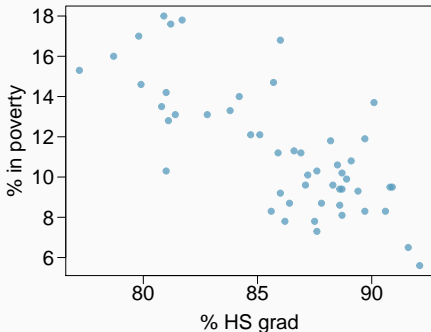
Response variable?

% in poverty

Explanatory variable?

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Response variable?

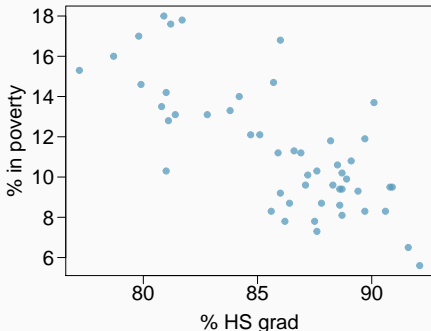
% in poverty

Explanatory variable?

% HS grad

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Response variable?

% in poverty

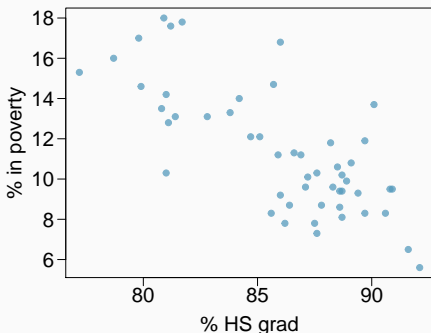
Explanatory variable?

% HS grad

Relationship?

Poverty vs. HS graduate rate

The *scatterplot* below shows the relationship between HS graduate rate in all 50 US states and DC and the % of residents who live below the poverty line (income below \$23,050 for a family of 4 in 2012).



Response variable?

% in poverty

Explanatory variable?

% HS grad

Relationship?

linear, negative, moderately strong

Quantifying the relationship

- *Correlation* describes the strength of the *linear* association between two variables.

Quantifying the relationship

- *Correlation* describes the strength of the *linear* association between two variables.
- It takes values between -1 (perfect negative) and +1 (perfect positive).

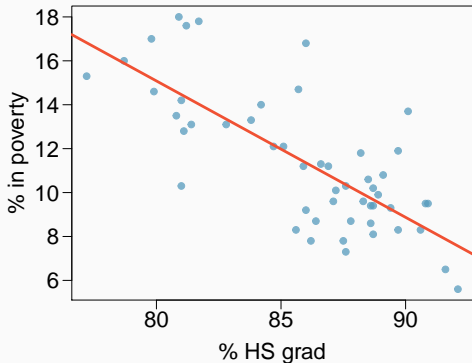
Quantifying the relationship

- *Correlation* describes the strength of the *linear* association between two variables.
- It takes values between -1 (perfect negative) and +1 (perfect positive).
- A value of 0 indicates no linear association.

Guessing the correlation

Which of the following is the best guess for the correlation between % in poverty and % HS grad?

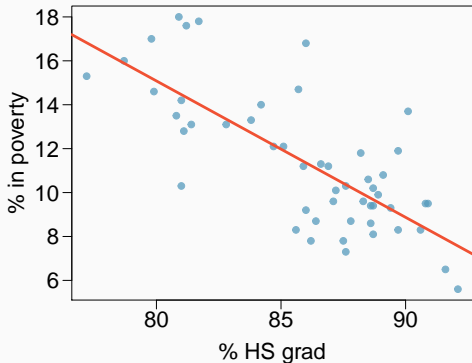
- (a) 0.6
- (b) -0.75
- (c) -0.1
- (d) 0.02
- (e) -1.5



Guessing the correlation

Which of the following is the best guess for the correlation between % in poverty and % HS grad?

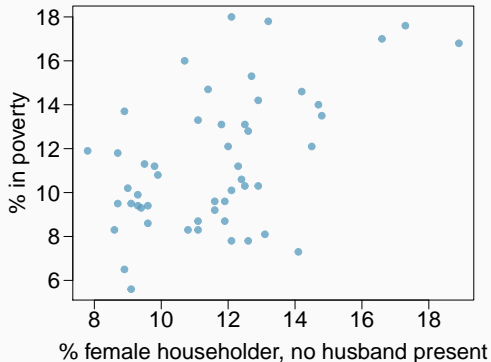
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Guessing the correlation

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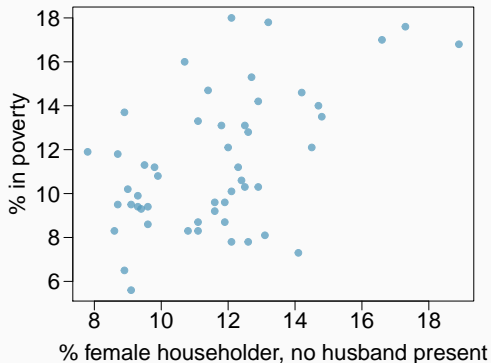
- (a) 0.1
- (b) -0.6
- (c) -0.4
- (d) 0.9
- (e) 0.5



Guessing the correlation

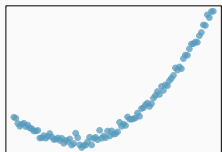
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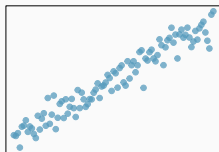


Assessing the correlation

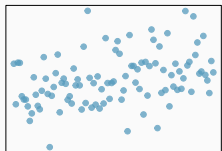
Which of the following is has the strongest correlation, i.e. correlation coefficient closest to +1 or -1?



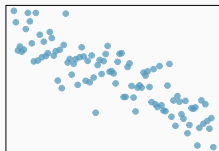
(a)



(b)



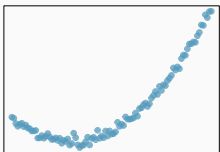
(c)



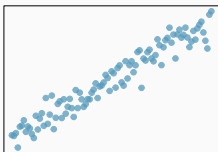
(d)

Assessing the correlation

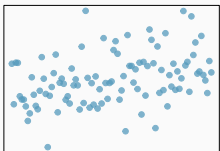
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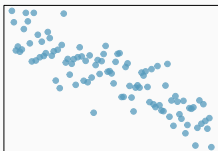
(a)



(b)



(c)



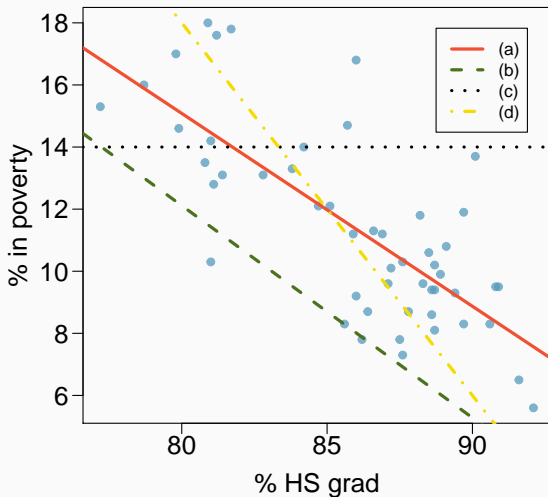
(d)

(b) →
correlation
means linear
association

Fitting a line by least squares regression

Eyeballing the line

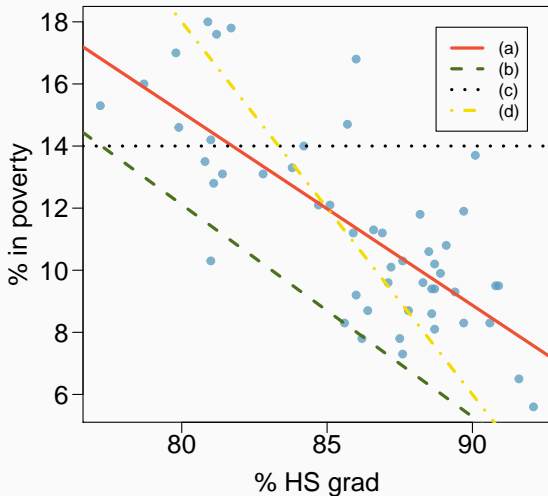
Which of the following appears to be the line that best fits the linear relationship between % in poverty and % HS grad? Choose one.



Eyeballing the line

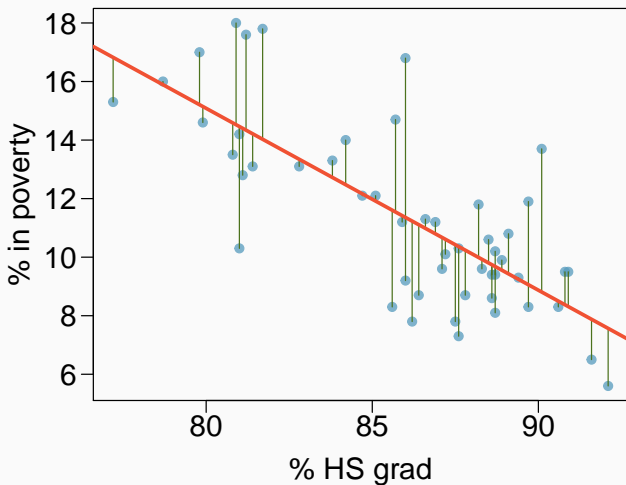
Which of the following appears to be the line that best fits the linear relationship between % in poverty and % HS grad? Choose one.

(a)



Residuals

Residuals are the leftovers from the model fit: $\text{Data} = \text{Fit} + \text{Residual}$

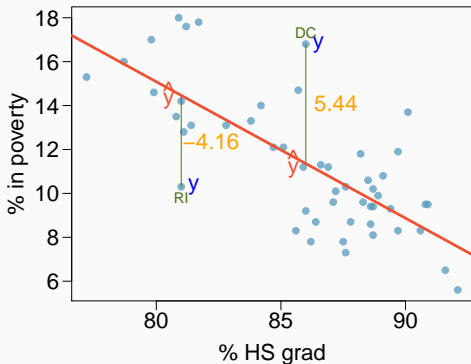


Residuals (cont.)

Residual

Residual is the difference between the observed (y_i) and predicted \hat{y}_i .

$$e_i = y_i - \hat{y}_i$$

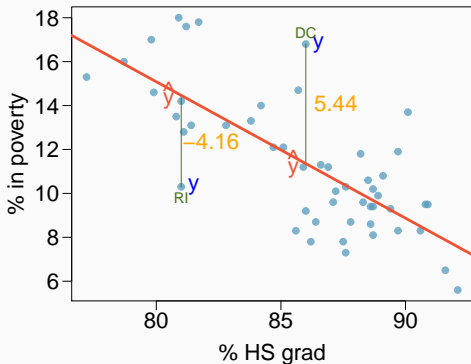


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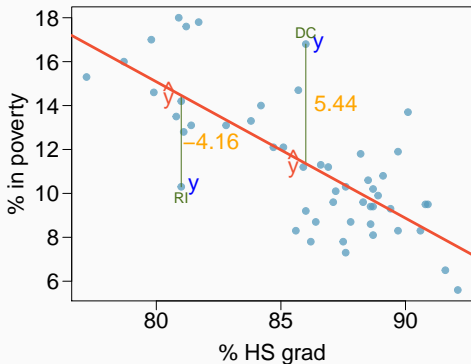
- % living in poverty in DC is 5.44% more than predicted.

Residuals (cont.)

Residual

Residual is the difference between the observed (y_i) and predicted \hat{y}_i .

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- % living in poverty in DC is 5.44% more than predicted.
- % living in poverty in RI is 4.16% less than predicted.

A measure for the best line

- We want a line that has small residuals:

A measure for the best line

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 1. Option 1: Minimize the sum of magnitudes (absolute values) of residuals

$$|e_1| + |e_2| + \cdots + |e_n|$$

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- Why least squares?
 1. Most commonly used

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 1. Most commonly used
 2. Easier to compute by hand and using software

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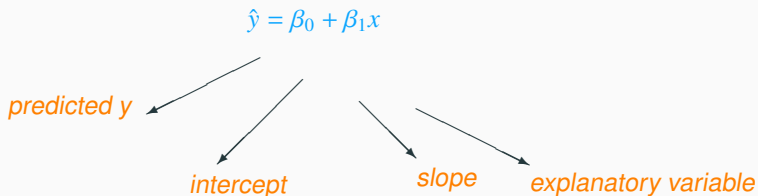
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$$e_1^2 + e_2^2 + \cdots + e_n^2$$

- Why least squares?
 1. Most commonly used
 2. Easier to compute by hand and using software
 3. In many applications, a residual twice as large as another is usually more than twice as bad

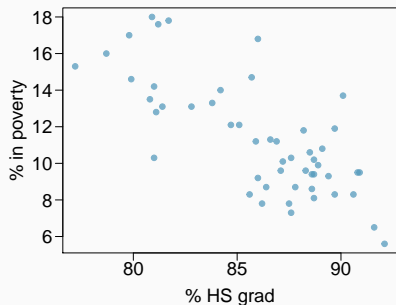
The least squares line



Notation:

- Intercept:
 - Parameter: β_0
 - Point estimate: b_0
- Slope:
 - Parameter: β_1
 - Point estimate: b_1

Given...



	% HS grad (x)	% in poverty (y)
mean	$\bar{x} = 86.01$	$\bar{y} = 11.35$
sd	$s_x = 3.73$	$s_y = 3.1$
correlation		$R = -0.75$

Slope

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The slope of the regression can be calculated as

$$b_1 = \frac{s_y}{s_x}R$$

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In context...

$$b_1 = \frac{3.1}{3.73} \times -0.75 = -0.62$$

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$$b_1 = \frac{s_y}{s_x}R$$

In context...

$$b_1 = \frac{3.1}{3.73} \times -0.75 = -0.62$$

Interpretation

For each additional % point in HS graduate rate, we would expect the % living in poverty to be lower on average by 0.62% points.

Intercept

Intercept

The intercept is where the regression line intersects the y -axis. The calculation of the intercept uses the fact that a regression line always passes through (\bar{x}, \bar{y}) .

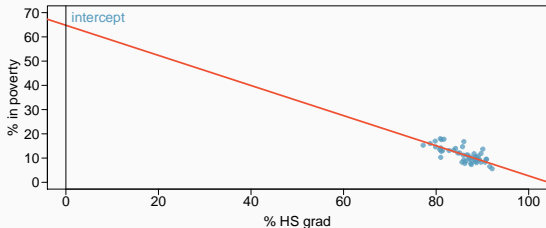
$$b_0 = \bar{y} - b_1\bar{x}$$

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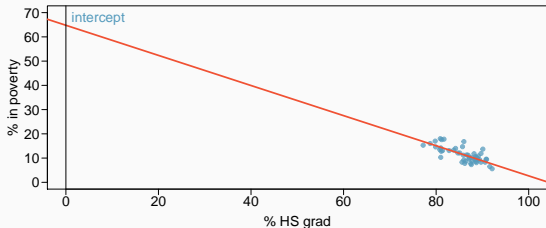


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$$b_0 = \bar{y} - b_1\bar{x}$$



$$\begin{aligned} b_0 &= 11.35 - (-0.62) \times 86.01 \\ &= 64.68 \end{aligned}$$

Which of the following is the correct interpretation of the intercept?

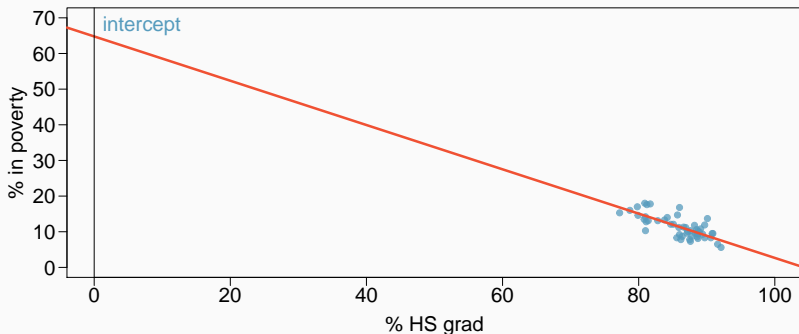
- (a) For each % point increase in HS graduate rate, % living in poverty is expected to increase on average by 64.68%.
- (b) For each % point decrease in HS graduate rate, % living in poverty is expected to increase on average by 64.68%.
- (c) Having no HS graduates leads to 64.68% of residents living below the poverty line.
- (d) States with no HS graduates are expected on average to have 64.68% of residents living below the poverty line.
- (e) In states with no HS graduates % living in poverty is expected to increase on average by 64.68%.

Which of the following is the correct interpretation of the intercept?

- (a) For each % point increase in HS graduate rate, % living in poverty is expected to increase on average by 64.68%.
- (b) For each % point decrease in HS graduate rate, % living in poverty is expected to increase on average by 64.68%.
- (c) Having no HS graduates leads to 64.68% of residents living below the poverty line.
- (d) *States with no HS graduates are expected on average to have 64.68% of residents living below the poverty line.*
- (e) In states with no HS graduates % living in poverty is expected to increase on average by 64.68%.

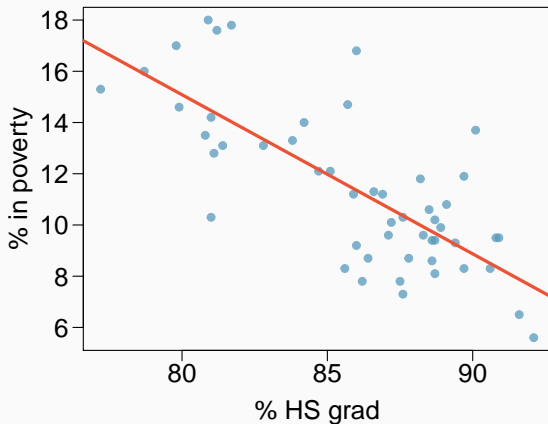
More on the intercept

Since there are no states in the dataset with no HS graduates, the intercept is of no interest, not very useful, and also not reliable since the predicted value of the intercept is so far from the bulk of the data.



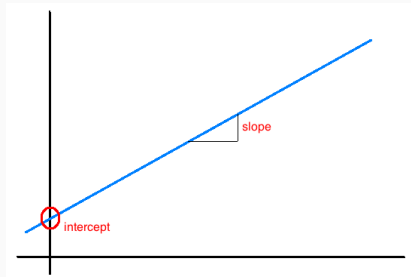
Regression line

$$\widehat{\% \text{ in poverty}} = 64.68 - 0.62 \% \text{ HS grad}$$



Interpretation of slope and intercept

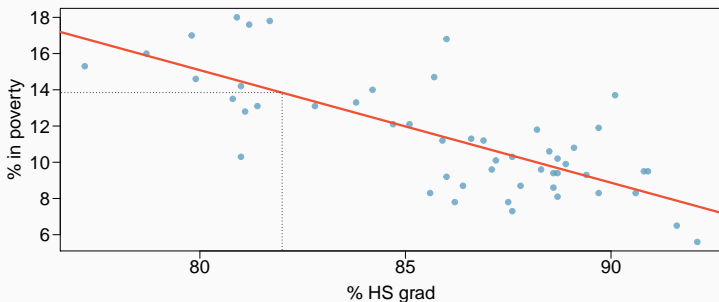
- **Intercept:** When $x = 0$, y is expected to equal the intercept.
- **Slope:** For each unit in x , y is expected to increase / decrease on average by the slope.



Note: These statements are not causal, unless the study is a randomized controlled experiment.

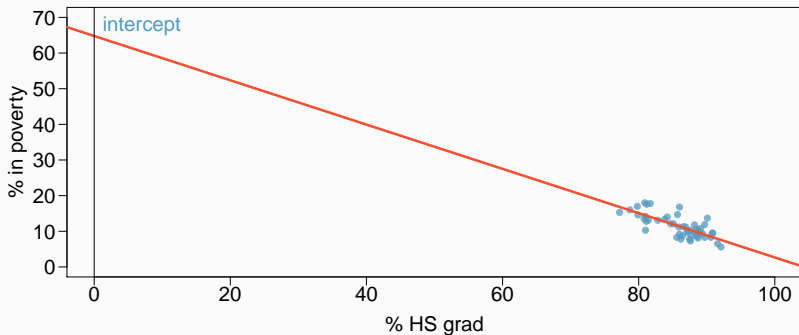
Prediction

- Using the linear model to predict the value of the response variable for a given value of the explanatory variable is called *prediction*, simply by plugging in the value of x in the linear model equation.
- There will be some uncertainty associated with the predicted value.

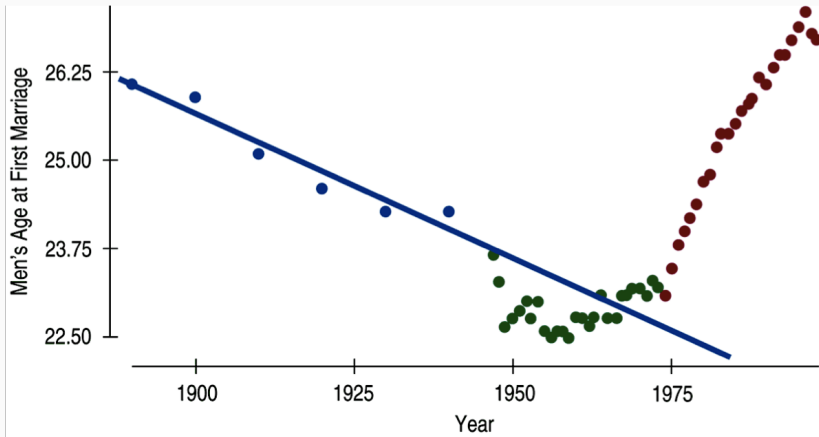


Extrapolation

- Applying a model estimate to values outside of the realm of the original data is called *extrapolation*.
- Sometimes the intercept might be an extrapolation.



Examples of extrapolation



Examples of extrapolation

BBC NEWS

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Women 'may outspurt men by 2156'

Women sprinters may be outrunning men in the 2156 Olympics if they continue to close the gap at the rate they are doing, according to scientists.



Women are set to become the dominant sprinters

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England
Northern Ireland
Scotland
Wales
UK Politics
Education
Magazine
Business
Health
Science & Environment
Technology
Entertainment
Also in the news

An Oxford University study found that women are running faster than they have ever done over 100m.

At their current rate of improvement, they should overtake men within 150 years, said Dr Andrew Tatem.

The study, comparing winning times for the Olympic 100m since 1900, is published in the journal Nature.

However, former British Olympic sprinter Derek Redmond told the BBC: "I find it difficult to believe.

"I can see the gap closing between men and women but I can't necessarily see it being overtaken because mens' times are also going to improve."

Examples of extrapolation

Momentous sprint at the 2156 Olympics?

Women sprinters are closing the gap on men and may one day overtake them.

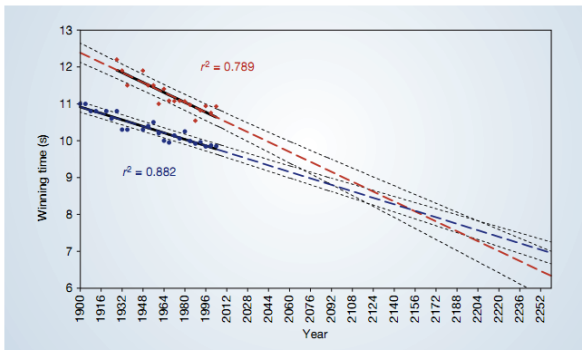


Figure 1 The winning Olympic 100-metre sprint times for men (blue points) and women (red points), with superimposed best-fit linear regression lines (solid black lines) and coefficients of determination. The regression lines are extrapolated (broken blue and red lines for men and women, respectively) and 95% confidence intervals (dotted black lines) based on the available points are superimposed. The projections intersect just before the 2156 Olympics, when the winning women's 100-metre sprint time of 8.079 s will be faster than the men's at 8.098 s.

Conditions for the least squares line

1. Linearity

Conditions for the least squares line

1. Linearity
2. Nearly normal residuals

Conditions for the least squares line

1. Linearity
2. Nearly normal residuals
3. Constant variability

Conditions: (1) Linearity

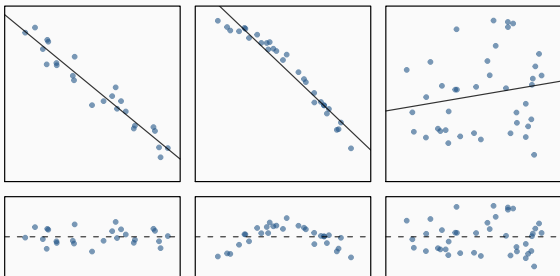
- The relationship between the explanatory and the response variable should be linear.

Conditions: (1) Linearity

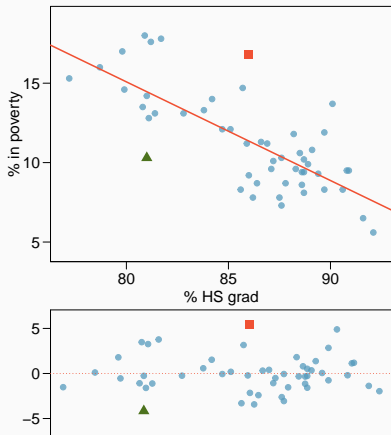
- The relationship between the explanatory and the response variable should be linear.
- Methods for fitting a model to non-linear relationships exist, but are beyond the scope of this class. If this topic is of interest, an Online Extra is available on openintro.org covering new techniques.

Conditions: (1) Linearity

- The relationship between the explanatory and the response variable should be linear.
- Methods for fitting a model to non-linear relationships exist, but are beyond the scope of this class. If this topic is of interest, an Online Extra is available on openintro.org covering new techniques.
- Check using a scatterplot of the data, or a *residuals plot*.



Anatomy of a residuals plot



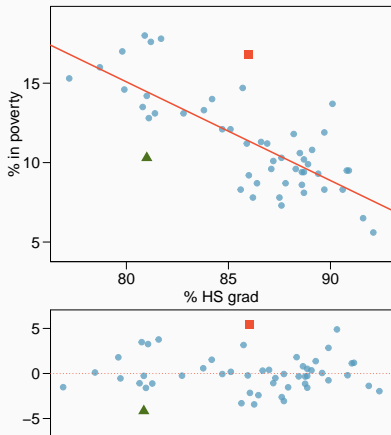
▲ RI:

$$\% \text{ HS grad} = 81 \quad \% \text{ in poverty} = 10.3$$

$$\% \text{ in } \widehat{\text{poverty}} = 64.68 - 0.62 * 81 = 14.46$$

$$\begin{aligned} e &= \% \text{ in poverty} - \% \text{ in } \widehat{\text{poverty}} \\ &= 10.3 - 14.46 = -4.16 \end{aligned}$$

Anatomy of a residuals plot



▲ RI:

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$$\% \text{ in } \widehat{\text{poverty}} = 64.68 - 0.62 * 81 = 14.46$$

$$\begin{aligned} e &= \% \text{ in poverty} - \% \text{ in } \widehat{\text{poverty}} \\ &= 10.3 - 14.46 = -4.16 \end{aligned}$$

■ DC:

$$\% \text{ HS grad} = 86 \quad \% \text{ in poverty} = 16.8$$

$$\% \text{ in } \widehat{\text{poverty}} = 64.68 - 0.62 * 86 = 11.36$$

$$\begin{aligned} e &= \% \text{ in poverty} - \% \text{ in } \widehat{\text{poverty}} \\ &= 16.8 - 11.36 = 5.44 \end{aligned}$$

Conditions: (2) Nearly normal residuals

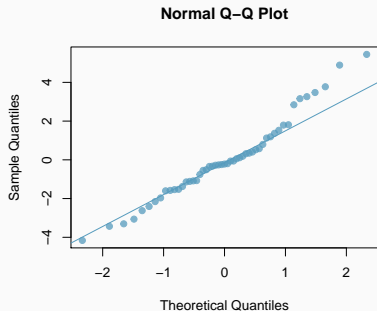
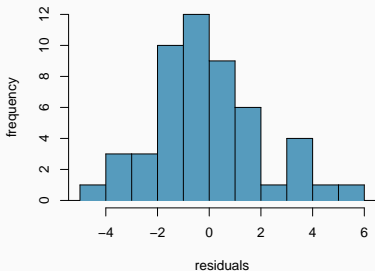
- The residuals should be nearly normal.

Conditions: (2) Nearly normal residuals

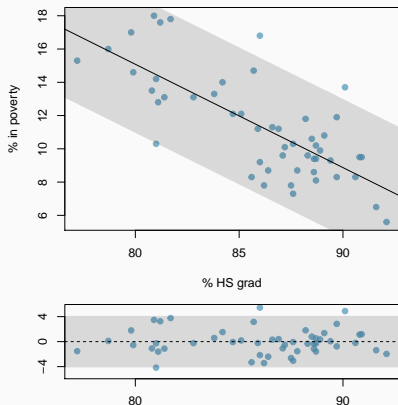
- The residuals should be nearly normal.
- This condition may not be satisfied when there are unusual observations that don't follow the trend of the rest of the data.

Conditions: (2) Nearly normal residuals

- The residuals should be nearly normal.
- This condition may not be satisfied when there are unusual observations that don't follow the trend of the rest of the data.
- Check using a histogram or normal probability plot of residuals.

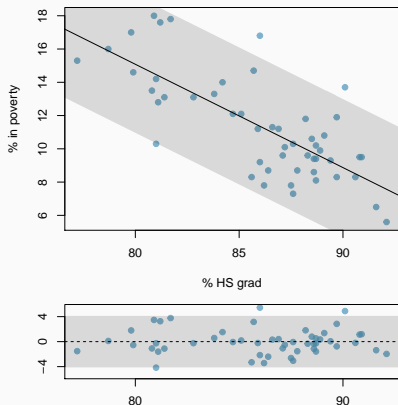


Conditions: (3) Constant variability



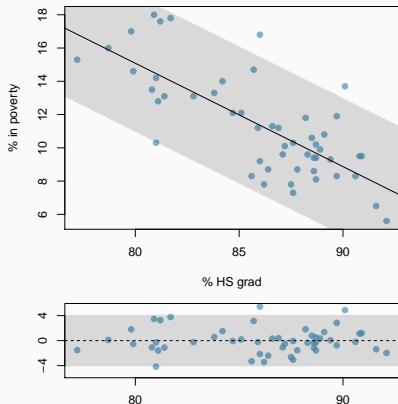
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Conditions: (3) Constant variability



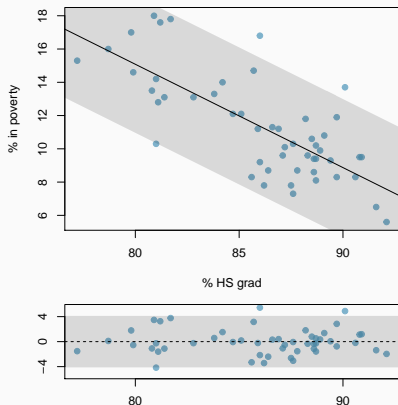
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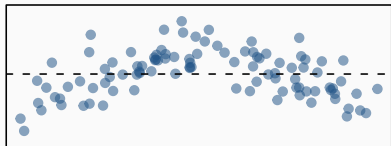
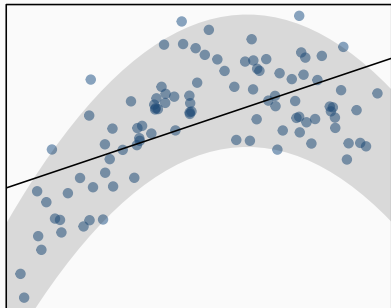


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Checking conditions

What condition is this linear model obviously violating?

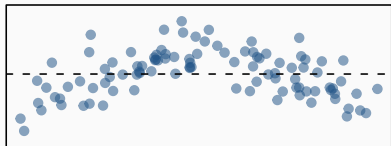
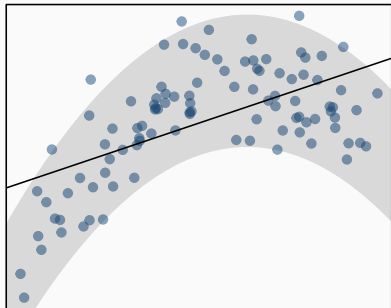
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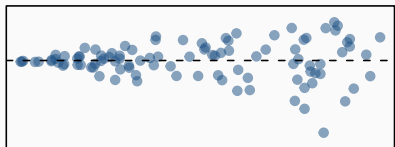
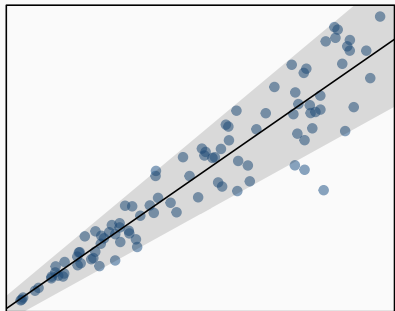
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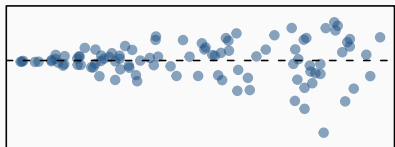
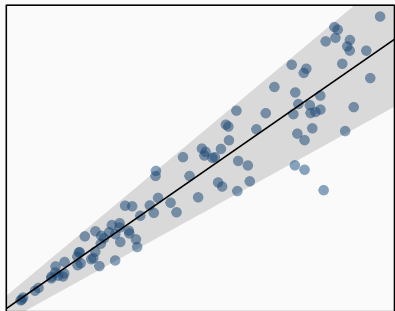
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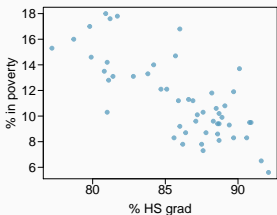
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- The remainder of the variability is explained by variables not included in the model or by inherent randomness in the data.
- For the model we've been working with, $R^2 = -0.62^2 = 0.38$.

Interpretation of R^2

Which of the below is the correct interpretation of $R = -0.62$, $R^2 = 0.38$?

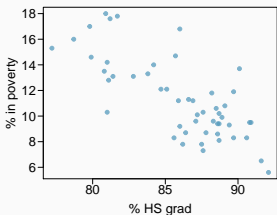
- (a) 38% of the variability in the % of HG graduates among the 51 states is explained by the model.
- (b) 38% of the variability in the % of residents living in poverty among the 51 states is explained by the model.
- (c) 38% of the time % HS graduates predict % living in poverty correctly.
- (d) 62% of the variability in the % of residents living in poverty among the 51 states is explained by the model.



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Poverty vs. region (east, west)

$$\widehat{poverty} = 11.17 + 0.38 \times west$$

- Explanatory variable: region, *reference level*: east
- *Intercept*: The estimated average poverty percentage in eastern states is 11.17%

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- *Slope*: The estimated average poverty percentage in western states is 0.38% higher than eastern states.
 - Then, the estimated average poverty percentage in western states is $11.17 + 0.38 = 11.55\%$.
 - This is the value we get if we plug in *1* for the explanatory variable

Poverty vs. region (northeast, midwest, west, south)

Which region (northeast, midwest, west, or south) is the reference level?

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	9.50	0.87	10.94	0.00
region4midwest	0.03	1.15	0.02	0.98
region4west	1.79	1.13	1.59	0.12
region4south	4.16	1.07	3.87	0.00

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Linear models in the tidyverse

Basic method for linear fitting in R

- We use the `sim1` dataset loaded via `library(modelr)` for the following demonstration.

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```
library(modelr)
```

The first few rows of the dataset are:

```
head(sim1)
```

x	y
1	4.199913
1	7.510634
1	2.125473
2	8.988857
2	10.243105
2	11.296823

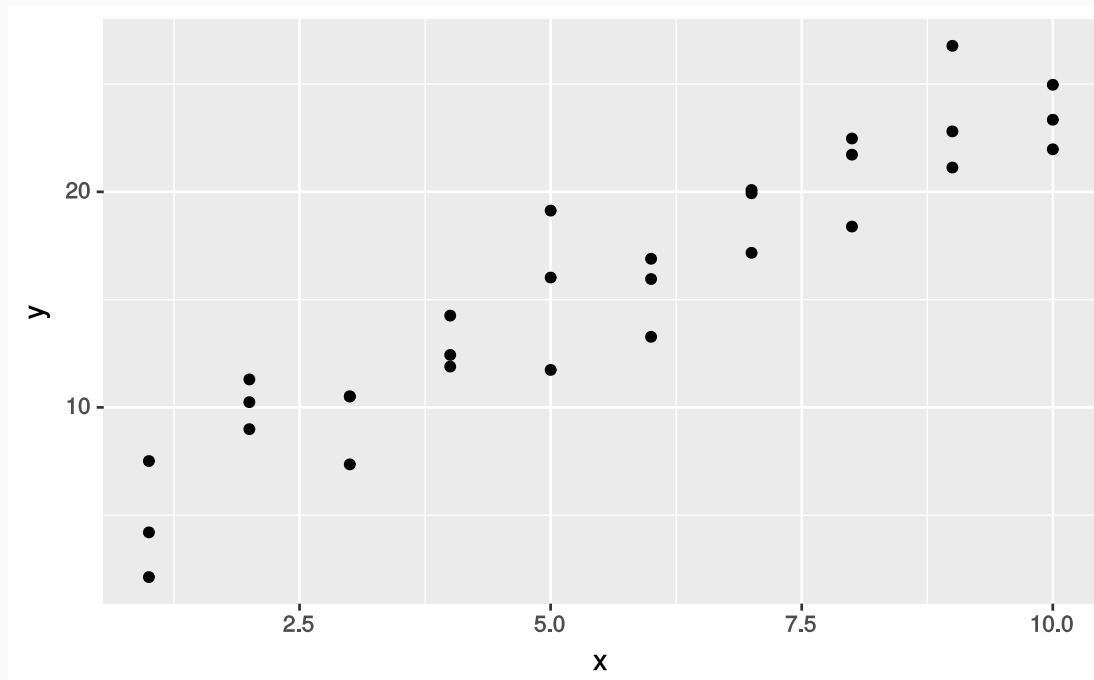
Visualize the dataset

- Let's look at a scatterplot of the dataset:

Visualize the dataset

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```
ggplot(sim1) +  
  geom_point(aes(x, y))
```



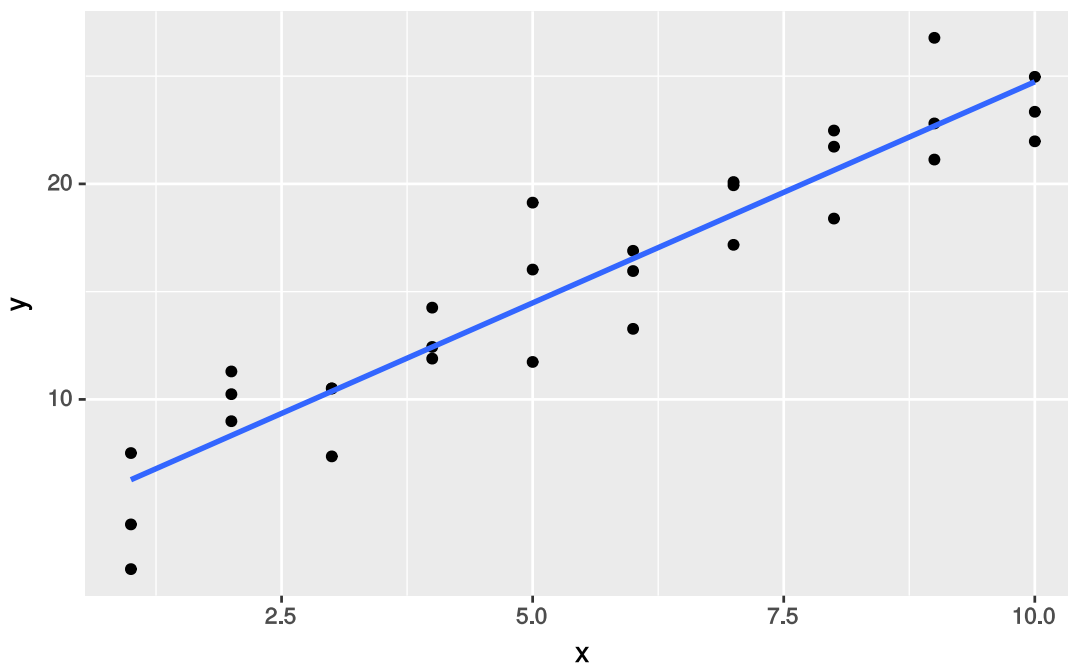
ggplot2 can create linear models

- Remember `geom_smooth`? We can just this to create linear models with `ggplot2`:

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```
ggplot(sim1) +  
  geom_point(mapping = aes(x = x, y = y)) +  
  geom_smooth(mapping = aes(x = x, y = y), method = "lm", se = FALSE)
```



R's standard method

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```
sim1_mod <- lm(y ~ x, data = sim1)
```

Summary of linear model

- For a general report about the model, use `summary()`:

Summary of linear model

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```
summary(sim1_mod)
```

```
##
## Call:
## lm(formula = y ~ x, data = sim1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.1469 -1.5197  0.1331  1.4670  4.6516
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.2208     0.8688   4.858 4.09e-05 ***
## x             2.0515     0.1400  14.651 1.17e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.203 on 28 degrees of freedom
## Multiple R-squared:  0.8846,    Adjusted R-squared:  0.8805
## F-statistic: 214.7 on 1 and 28 DF,  p-value: 1.173e-14
```

Reporting the model

- We report the model as:

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$$y = 2.0515x + 4.2208$$

Method for plotting our model

- The following is a basic recipe for visualizing our models

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- Create a series of `x` values with `data_grid()`:

```
grid <- data_grid(sim1, x)
```


Method for plotting our model

- The following is a basic recipe for visualizing our models
- Create a series of `x` values with `data_grid()`:

```
grid <- data_grid(sim1, x)
```

x

1

2

3

4

5

6

Extract predictions and residuals

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```
grid2 <- add_predictions(grid, sim1_mod)
```

- Use `add_residuals()` to extract the residuals from your fit.

```
sim1_resid <- add_residuals(sim1, sim1_mod)
```

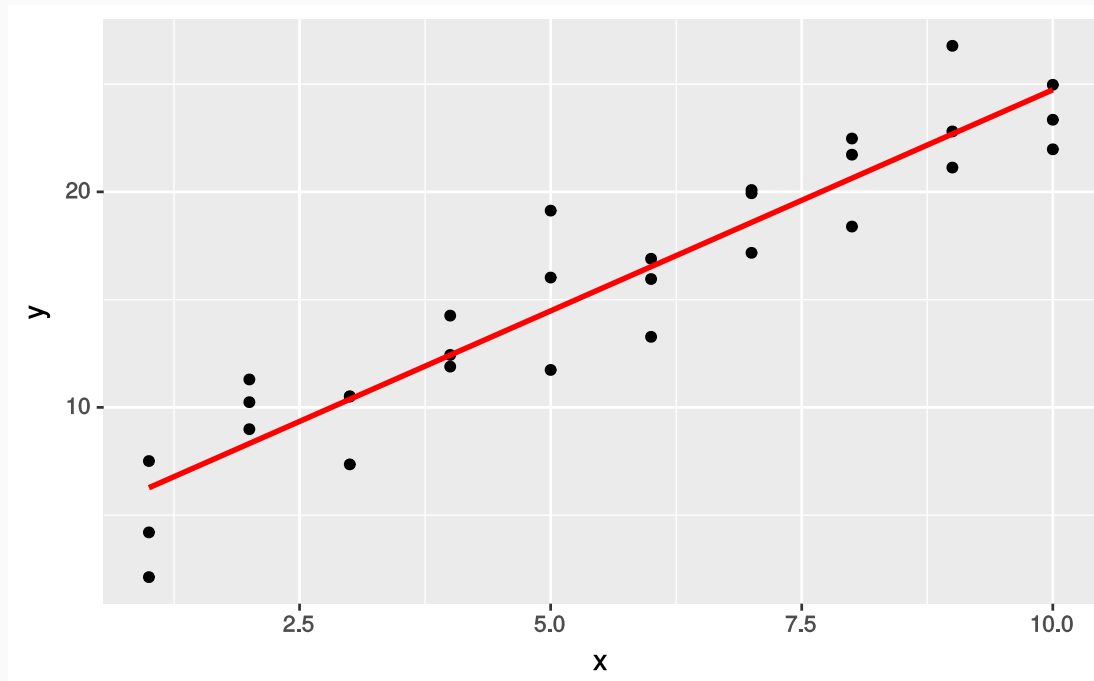
Visualize the full model

- Create a plot:

Visualize the full model

- Create a plot:

```
ggplot(sim1) +  
  geom_point(aes(x = x, y = y)) +  
  geom_line(aes(x = x, y = pred), data = grid2, color = "red", size = 1)
```



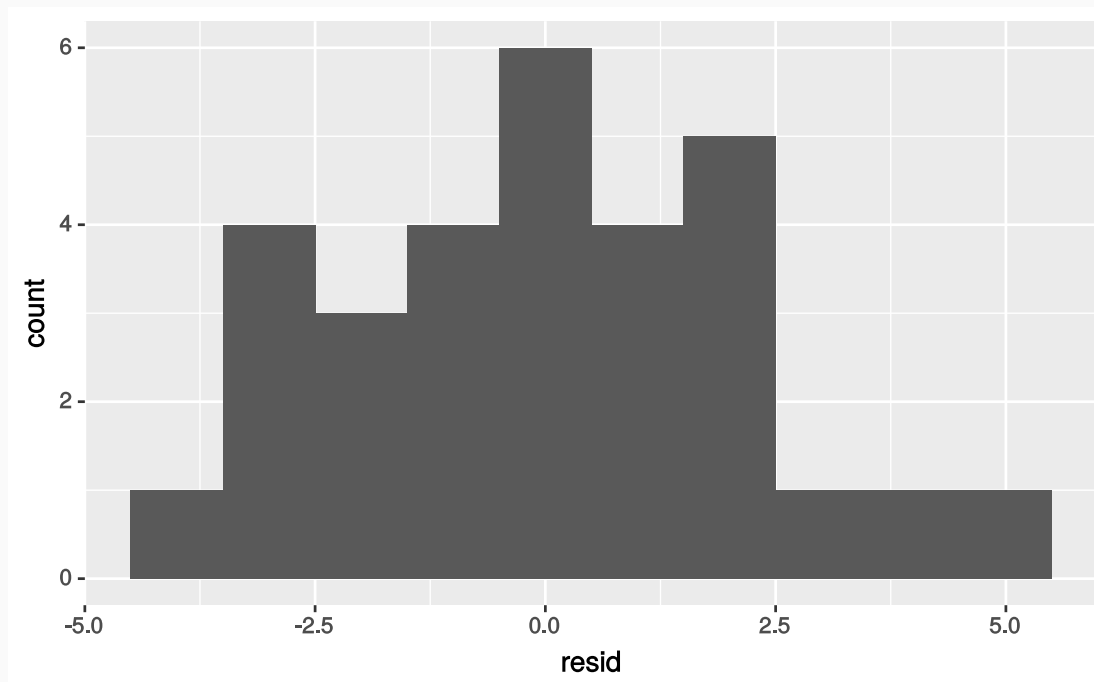
Inspect the residuals

- Use `geom_histogram()` to inspect the absolute residuals.

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```
ggplot(sim1_resid) +  
  geom_histogram(aes(x = resid), binwidth = 1)
```



Are the residuals normal?

- The residuals should be nearly normal.

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- A good test for normal residuals is a Q-Q plot:

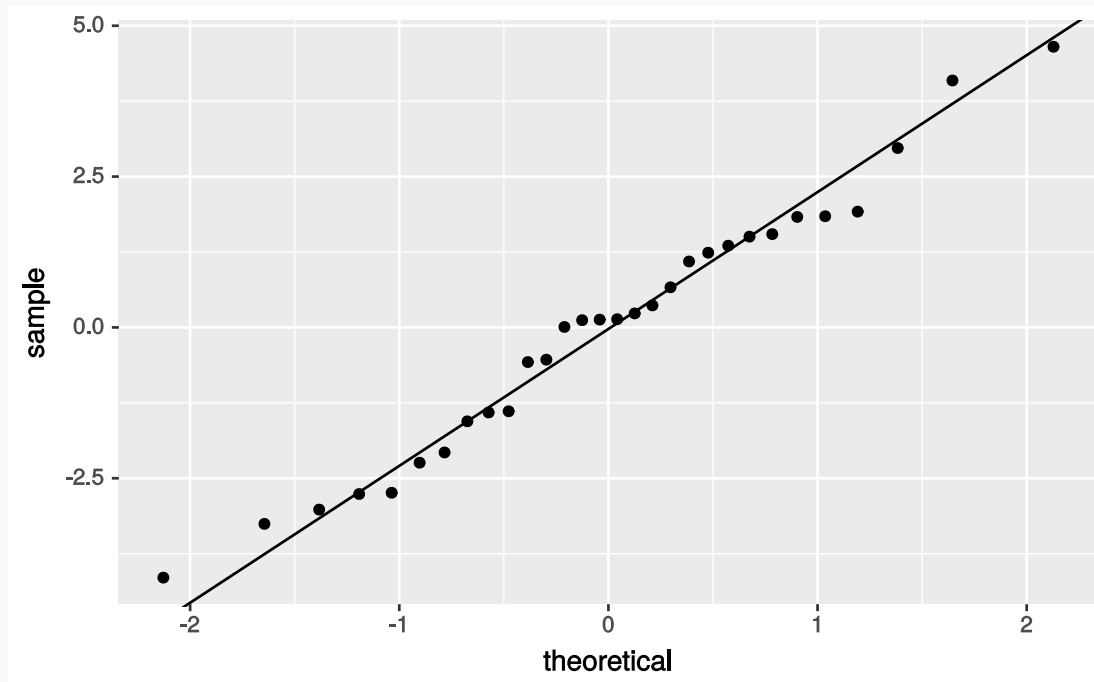
Are the residuals normal?

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```
qq_x <- qnorm(p = c(0.25, 0.75))
qq_y <- quantile(x = pull(sim1_resid, resid), probs = c(0.25, 0.75), type = 1)
qq_slope <- diff(qq_y) / diff(qq_x)
qq_int <- pluck(qq_y, 1) - qq_slope * pluck(qq_x, 1)
ggplot(sim1_resid) +
  geom_qq(aes(sample = resid)) +
  geom_abline(intercept = qq_int, slope = qq_slope)
```

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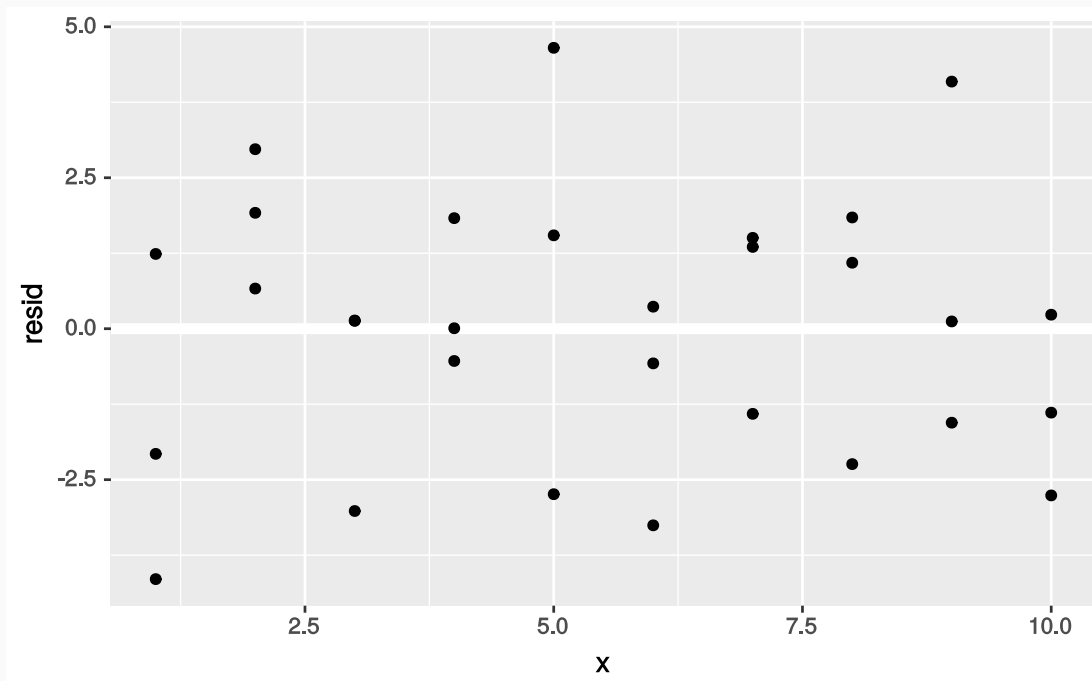
Residual spread

- Inspect the residual spread as a function of x to check whether the variability is constant or not:

Residual spread

- Inspect the residual spread as a function of `x` to check whether the variability is constant or not:

```
ggplot(sim1_resid) +  
  geom_ref_line(h = 0) +  
  geom_point(aes(x = x, y = resid))
```



Credits

`modelr` package examples adapted from content in chapters 23.2 and 23.3 of *R for Data Science* by Hadley Wickham and Garrett Golemund and made available under the [CC BY-NC-ND 3.0 license](#).

Content in the slides with blue headers adapted from the chapter 7 [OpenIntro Statistics slides](#) developed by Mine Çetinkaya-Rundel and made available under the [CC BY-SA 3.0 license](#).